SwInBee 2026

Name: ____ **Score:** _____

Instructions

1. Duration: 50 minutes.
2. Record your answers on this answer sheet.
3. No materials allowed besides pens and pencils. Paper will be supplied for rough working.
4. No partial marks awarded. This includes the "+ C" for indefinite integrals: if an appropriate constant is not included then you will get zero.

SOLUTIONS — not for distribution

Integrals

1. $\displaystyle\int \frac{x^{2025}}{1+x^{2026}}\,dx$

   $\displaystyle= \frac{1}{2026}\ln\left(1+x^{2026}\right)+C$

   $1+x^{2026}>0$ *always, so* $|\cdot|$ *is unnecessary but harmless.*

   Method: substitute $u = 1+x^{2026}$.

2. $\displaystyle\int \frac{e^{2x}-1}{e^x}\,dx$

   $= e^x + e^{-x} + C$

   $= 2\cosh x + C$

   Method: split the fraction into $e^x - e^{-x}$.

3. $\displaystyle\int \sin x\, e^{\cos x}\,dx$

   $= -e^{\cos x} + C$

   Method: substitute $u = \cos x$.

4. $\displaystyle\int \frac{x^3-1}{x-1}\,dx$

   $\displaystyle= \frac{x^3}{3}+\frac{x^2}{2}+x+C$

   Method: $x^3-1=(x-1)(x^2+x+1)$ — the integrand is just x^2+x+1.

5. $\displaystyle\int \sin^2(3x)\,dx$

   $\displaystyle= \frac{x}{2}-\frac{1}{12}\sin(6x)+C$

   $\displaystyle= \frac{x}{2}-\frac{1}{6}\sin(3x)\cos(3x)+C$

   Method: power reduction, $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$.

6. $\displaystyle\int \frac{dx}{x^2+6x+13}$

$$= \frac{1}{2}\arctan\left(\frac{x+3}{2}\right) + C$$

Method: complete the square: $x^2+6x+13 = (x+3)^2+4$.

7. $\displaystyle\int x^2 e^{-x}\,dx$

$$= -\left(x^2+2x+2\right)e^{-x} + C$$

Method: integration by parts twice (or tabular integration).

8. $\displaystyle\int \sin(\ln x)\,dx$

$$= \frac{x}{2}\left(\sin(\ln x) - \cos(\ln x)\right) + C$$

Method: substitute $x = e^u$, or integrate by parts twice and solve for the integral.

9. $\displaystyle\int \frac{x^2}{x^2-4}\,dx$

$$= x + \ln|x-2| - \ln|x+2| + C$$

$$= x + \ln\left|\frac{x-2}{x+2}\right| + C$$

$$= x - 2\,\mathrm{artanh}\left(\frac{x}{2}\right) + C$$

all three agree; $\ln$ *without* $|\cdot|$ *is also fine.*

Method: divide first: the integrand is $1 + \dfrac{4}{x^2-4}$, then partial fractions.

10. $\displaystyle\int \frac{\sqrt{x}}{1+x}\,dx$

$$= 2\sqrt{x} - 2\arctan\sqrt{x} + C$$

Method: substitute $t = \sqrt{x}$, giving $2\int \frac{t^2}{1+t^2}\,dt$.

11. $\displaystyle\int \frac{dx}{1+\sin x}$

$$= \tan x - \sec x + C$$

$$= \frac{\sin x - 1}{\cos x} + C$$

$$= \frac{-2}{1+\tan(x/2)} + C$$

the three differ by constants — all correct.

Method: multiply above and below by $1-\sin x$; or Weierstrass $t = \tan(x/2)$.

12. $\displaystyle\int x\arctan x\,dx$

$$= \frac{x^2+1}{2}\arctan x - \frac{x}{2} + C$$

$$= \frac{x^2}{2}\arctan x - \frac{x}{2} + \frac{1}{2}\arctan x + C$$

Method: parts with $u = \arctan x$, then $\dfrac{x^2}{1+x^2} = 1 - \dfrac{1}{1+x^2}$.

13. $\displaystyle\int \cos^5 x\,dx$

$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$$

Method: save one $\cos x$ for du; $\cos^4 x = (1-\sin^2 x)^2$.

14. $\displaystyle\int \frac{xe^x}{(1+x)^2}\,dx$

$$= \frac{e^x}{1+x} + C$$

Method: write $x = (1+x) - 1$ — the integrand is exactly $\frac{d}{dx}\left[\frac{e^x}{1+x}\right]$.

15. $\displaystyle\int \frac{dx}{e^x + e^{-x}}$

$$= \arctan(e^x) + C$$

$$= -\arctan\left(e^{-x}\right) + C$$

$$= \frac{1}{2}\arctan(\sinh x) + C$$

all three differ only by constants. Watch for $\arctan(\sinh x)$ without *the* $\frac{1}{2}$ *— that is wrong.*

Method: multiply above and below by e^x, then $u = e^x$.

16. $\displaystyle\int_0^{\pi/2} \frac{\sin^{2026} x}{\sin^{2026} x + \cos^{2026} x}\,dx$

$$= \frac{\pi}{4} \quad \approx 0.785398163$$

Method: "king's rule": $x \mapsto \frac{\pi}{2} - x$ swaps sin and cos, so $2I = \int_0^{\pi/2} 1\,dx$. The exponent is irrelevant.

17. $\displaystyle\int_{-1}^{1} \left(x^5 e^{x^2} + \frac{x}{1+x^4} + x^2\right) dx$

$$= \frac{2}{3} \quad \approx 0.666666667$$

Method: the first two terms are odd, so they vanish over $[-1,1]$; only $\int_{-1}^{1} x^2\,dx$ survives.

18. $\displaystyle\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$

$$= \frac{\pi^2}{4} \quad \approx 2.467401100$$

Method: $x \mapsto \pi - x$ gives $I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$, then $u = \cos x$.

19. $\displaystyle\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

$$= \frac{e^x}{x} + C$$

neither term is elementary on its own — only the combination is. Computer algebra systems often return $\mathrm{Ei}(x)$ *here; that is not wrong, just unsimplified.*

Method: recognise $\frac{d}{dx}\left[\frac{e^x}{x}\right] = \frac{e^x}{x} - \frac{e^x}{x^2}$.

20. $\displaystyle\int \frac{x^2+1}{x^4+1}\,dx$

$$= \frac{1}{\sqrt{2}}\arctan\left(\frac{x^2-1}{x\sqrt{2}}\right) + C$$

equivalently $\frac{1}{\sqrt{2}}\arctan\left(\frac{x-1/x}{\sqrt{2}}\right) + C$. *Valid on each side of* $x = 0$ *separately.*

Method: divide above and below by x^2: $\dfrac{1+1/x^2}{x^2+1/x^2}$, then $u = x - \dfrac{1}{x}$, $u^2 + 2 = x^2 + 1/x^2$.