SwInBee 2025

Name: _____   **Score:** _____

Instructions

1. Duration: 50 minutes.

2. Record your answers on this answer sheet.

3. No materials allowed besides pens and pencils. Paper will be supplied for rough working.

4. No partial marks awarded. This includes the "+ C" for indefinite integrals: if an appropriate constant is not included then you will get zero.

Integrals

1. $\displaystyle\int x^{2025} - 2025^x \, dx$

   $= \dfrac{x^{2026}}{2026} - \dfrac{2025^x}{\ln 2025} + C$

2. $\displaystyle\int x^{2025} \ln x \, dx$

   $= \dfrac{x^{2026}}{2026} \left(\ln x - \dfrac{1}{2026} \right) + C$ Also OK to have $\ln |x|$

3. $\displaystyle\int \dfrac{x + 2025}{x - 2025} \, dx$

   $= x + 4050 \ln(x - 2025) + C$ Also OK to have $\ln |x - 2025|$

4. $\displaystyle\int_{-\infty}^{\infty} e^{-4x^2 + 8x - 5} \, dx$ *Hint*: you may use the fact that $\displaystyle\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}$

   $= \dfrac{\sqrt{\pi}}{2e}$

5. $\displaystyle\int \dfrac{x}{9 + 4x^4} \, dx$

   $= \dfrac{1}{12} \arctan(\dfrac{2x^2}{3}) + C$ Hint: let $u = x^2$

6. $\displaystyle\int \dfrac{1}{x(5x^2 + 4)} \, dx$

   $= -\dfrac{1}{8} \ln(\dfrac{4}{x^2} + 5) + C$

$$= \frac{1}{8} \ln(\frac{x^2}{4 + 5x^2}) + C$$
$$= \frac{1}{4} \ln(x) - \frac{1}{8} \ln(4 + 5x^2) + C$$

7. $\int \frac{x}{(2 + 3x^2)^2} \, dx$

$$= -\frac{1}{6(3x^2 + 2)} + C \qquad \text{Hint: Let } u = 2 + 3x^2$$

8. $\int \cos^4 x - \sin^4 x \, dx$

$$= \frac{1}{2} \sin(2x) + C$$
$$= \sin x \cos x + C$$

9. $\int_0^\infty \frac{dx}{1 + e^x}$
$$= \ln 2 \text{ (indefinite integral } - \ln(1 + e^{-x}) + C$$

10. $\int e^x \sin x \, dx$

$$= \frac{1}{2} e^x (\sin x - \cos x) + C$$

11. $\int \ln(2 + x^2) \, dx$

$$= x \ln(2 + x^2) - 2x + 2\sqrt{2} \arctan(x/\sqrt{2}) + C$$

12. $\int \frac{e^{-1/x^2}}{x^5} \, dx$

$$= \frac{1}{2} e^{-1/x^2} + \frac{1}{2x^2} e^{-1/x^2} + C$$

13. $\int_0^1 \frac{x^4(x - 1)^4}{x^2 + 1} \, dx$

$$= \frac{22}{7} - \pi$$

14. $\int \sinh x \cosh x \, dx$

$$= \cosh^2(x)/2 + C$$

$$= \sinh^2(x)/2 + C$$
$$= \frac{1}{4}e^{2x} + \frac{1}{4}e^{-2x} + C$$

15. $\displaystyle\int_0^1 \sum_{n=2}^{\infty} \frac{1}{(x+n)(x+n+1)}\,dx = \int_0^1 \left[\frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \frac{1}{(x+4)(x+5)} + \cdots \right] dx$

$$= \int_0^1 \frac{1}{x+2}\,dx$$
$$= \ln(3) - \ln(2)$$

16. $\displaystyle\int \frac{1}{x(\ln x)^2}\,dx$

$$= -\frac{1}{\ln x} + C$$

17. $\displaystyle\int \frac{x+4}{(x-1)^2}\,dx$

$$= -\frac{5}{x-1} + \ln(x-1) + C$$

18. $\displaystyle\int \frac{1}{(x^2-1)(x^2-4)}\,dx$

$$= \frac{1}{12}\left[2\ln(x+1) - 2\ln(x-1) - \ln(x+2) + \ln(x-2)\right] + C \qquad \text{Also fine } \ln|\cdot|$$

19. $\displaystyle\int \tan(3x)\,dx$

$$= -\frac{1}{3}\ln(\cos(3x)) + C \qquad \text{Also fine } \ln|\cdot|$$

20. $\displaystyle\int \frac{1}{x^3(x^2+1)}\,dx$

$$= -\frac{1}{2x^2} + \frac{1}{2}\ln(x^2+1) - \ln x + C \qquad \text{Also fine } \ln|\cdot|$$