SwInBee 2024

Name: _____ **Submission time:** _____ **Score:** _____

Instructions

1. Duration: 50 minutes.
2. Record your answers on this answer sheet.
3. No materials allowed besides pens and pencils. Paper will be supplied for rough working.
4. No partial marks awarded. This includes the "+ C" for indefinite integrals: if an appropriate constant is not included then you will get zero.
5. In the event of papers achieving the same score, the tie-breaker will be the order of submission, with earlier papers ranked higher.

Integrals

1. $\int \sin(2x)\cos(2x)\,dx$
   $= -\cos(4x)/8 + C$

2. $\int x\,7^{x^2}\,dx$
   $= \frac{1}{2}\int 7^{x^2} d(x^2) = \frac{1}{2\ln(7)} 7^{x^2} + C$

3. $\int \ln^2(x)\,dx$

   $= \{\ln^2(x) = u,\ \ dx = dv,\ \ du = \frac{2\ln(x)}{x},\ \ v = x\} = x\ln^2(x) - \int \frac{2x\ln(x)\,dx}{x}$
   $= \{\ln(x) = u,\ dx = dv,\ du = dx/x,\ v = x\}$
   $= x\ln^2(x) - 2\left(x\ln(x) - \int dx\right) = x\ln^2(x) - 2\left(x\ln(x) - x\right) + C$

4. $\int \frac{dx}{1+\sqrt{x}}$

   $= \{x = t^2,\ dx = 2tdt\} = \int \frac{2tdt}{1+t} = 2\int \left(1 - \frac{1}{1+t}\right) dt$
   $= 2t - 2\ln(1+t) = 2\sqrt{x} - 2\ln(1+\sqrt{x}) + C$

5. $\int \frac{\sqrt{x^2-1}}{x}\,dx$

$= \{\sqrt{x^2-1} = t,\ \frac{xdx}{\sqrt{x^2-1}} = \frac{xdx}{t} = dt\} = \int \frac{t}{x}\frac{tdt}{x}$
$= \int \frac{t^2dt}{t^2+1} = \int \left(1 - \frac{1}{t^2+1}\right) dt = t - \arctan(t) = \sqrt{x^2-1} - \arctan(\sqrt{x^2-1}) + C$

6. $\int x\,dy$
$= xy + C$

7. $\int \frac{dx}{5 - 3\cos^2(x)}$

$= \{\tan(x) = t,\ \ x = \arctan(t),\ \ dx = \frac{dt}{1+t^2},\ \ \cos^2 x = \frac{1}{1+t^2}\}$
$= \int \frac{dt}{1+t^2}\frac{1}{5-3/(1+t^2)} = \int \frac{dt}{1+t^2}\frac{1+t^2}{5+5t^2-3} = \int \frac{dt}{2+5t^2} = \frac{1}{2}\int \frac{dt}{1+5t^2/2}$
$= \{\sqrt{5/2}t = y,\ \ \sqrt{5/2}dt = dy\} = \frac{1}{2}\sqrt{\frac{5}{2}}\int \frac{dy}{1+y^2} = \frac{1}{2}\sqrt{\frac{5}{2}}\arctan(y) + C$
$= \frac{1}{2}\sqrt{\frac{5}{2}}\arctan(\sqrt{5/2}\tan(x)) + C$

8. $\int \frac{4x+1}{1+x^2}dx$
$= 2\ln(x^2+1) + \arctan(x) + C$

9. $\int \sin^2 x\ \cos^2 x\,dx$
$= 1/32(4x - \sin(4x)) + C$

10. $\int \frac{4x}{\exp(2x+3)}dx$
$= -(1+2x)\exp(-2x-3) + C$

11. $\int \frac{2x^3}{5+3x^2}dx$
$= \frac{1}{9}\left(5 + 3x^2 - 5\ln(5+3x^2)\right) + C$

12. $\int \frac{dx}{\sqrt{x^2-2024}}$
$= \cosh^{-1}(x/\sqrt{2024}) + C$ Hint: $x = \sqrt{2024}\cosh t$.

13. $\int \dfrac{dx}{x\sqrt{x^2-1}}$
$= \cos^{-1}(1/x) + C$ Hint: $x = 1/\cos t$.

14. $\int \dfrac{\sqrt{x}}{\sqrt{x} - \sqrt[3]{x}}\, dx$

$$= 6\left[\frac{1}{6}x^{\frac{6}{6}} + \frac{1}{5}x^{\frac{5}{6}} + \frac{1}{4}x^{\frac{4}{6}} + \frac{1}{3}x^{\frac{3}{6}} + \frac{1}{2}x^{\frac{2}{6}} + \frac{1}{1}x^{\frac{1}{6}} + \ln\left(x^{\frac{1}{6}} - 1\right)\right] + C$$

15. $\int_{-\pi}^{\pi} x^{11}\sin^3(x^2)\, dx$
$= 0$

16. $\int \dfrac{3x+2}{x^2+3x+2}\, dx$
$= 4\ln(x+2) - \ln(x+1) + C$

17. $\int xe^x \cos x\, dx$
$\dfrac{1}{2}e^x((x-1)\sin x + x\cos x)$

18. $\int \dfrac{(\sin x - \cos x)(\sin x + \cos x)}{\sin x \cos x}\, dx$
$-\ln(\sin(2x)) + C = -\ln(\cos x) - \ln(\sin x) + C$

19. $\int \dfrac{\ln x}{x}\, dx$
$= \ln^2(x)/2 + C$

20. $\int_0^1 \dfrac{x^4(1-x)^4}{1+x^2}\, dx$
$= \dfrac{22}{7} - \pi$