SwInBee 2023

Name: ______________________ **Submission time:** ________ **Score:** _____

Instructions

1. Duration: 1 hour.
2. Record your answers on this answer sheet.
3. No materials allowed besides pens and pencils. Paper will be supplied for rough working.
4. No partial marks awarded. This includes the "+ C" for indefinite integrals: if an appropriate constant is not included then you will get zero.
5. In the event of papers achieving the same score, the tie-breaker will be the order of submission, with earlier papers ranked higher.

Integrals

1. $\displaystyle\int_0^\pi (2x+\pi)^{100}\,dx$

   $\displaystyle\frac{1}{202}((3\pi)^{101} - \pi^{101})$

2. $\displaystyle\int e^{-3x}(1+2e^{-x})^2\,dx$

   $\displaystyle\int e^{-3x}(1+2e^{-x})^2 dx = \frac{-4e^{-5x}}{5} - e^{-4x} - \frac{e^{-3x}}{3} + C$

3. $\displaystyle\int_0^{1/2} \sqrt{1-4x^2}\,dx$

   $\displaystyle\int_0^{1/2} \sqrt{1-4x^2} dx = \pi/8$, Hint substitute: $x = \frac{\sin\theta}{2}$ and use $\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$

   $\displaystyle\int \sqrt{1-4x^2}\,dx = x\sqrt{1-4x^2}/2 + \arcsin(2x)/4 + C$

4. $\displaystyle\int \sin^3 x\,dx$

   $\displaystyle\int \sin^3 x dx = \frac{\cos^3 x}{3} - \cos x + C = \frac{\cos 3x}{12} - \frac{3\cos x}{4} + C$

5. $\displaystyle\int \sin x\ \cos x\,dx$

   $\displaystyle-\frac{1}{4}x\cos(2x) + C$

6. $\displaystyle\int e^x \sin x \, dx$

$\dfrac{1}{2}e^x(\sin x - \cos x) + C$

7. $\displaystyle\int x^2 e^x \cos x \, dx$

$\dfrac{1}{2}e^x x^2(\cos(x) + \sin(x)) - e^x x \sin(x) + \dfrac{1}{2}e^x(\sin(x) - \cos(x)) + C$

8. $\displaystyle\int \ln\sqrt{1+\sqrt{1+x}} \, dx$

$-x/4 + \sqrt{1+x}/2 + x\ln(1+\sqrt{1+x})/2 + C$

9. $\displaystyle\int \frac{1}{1-\tan^2 x} \, dx$

$x/2 - \ln(\cos x - \sin x)/4 + \ln(\cos x + \sin x)/4 + C$

10. $\displaystyle\int x^2 \arcsin x \, dx$

$x^3 \arcsin x/3 + (x^2+2)\sqrt{1-x^2}/9 + C$

11. $\displaystyle\int \frac{1}{(x+1)(x+2)(x+3)} \, dx$

$\dfrac{1}{2}\left[\ln(x+1) - 2\ln(x+2) + \ln(x+3)\right] + C$

12. $\displaystyle\int x \sin x \cos x \, dx$

$\dfrac{1}{8}\sin(2x) - \dfrac{1}{4}\cos(2x) + C$

13. $\displaystyle\int x^2 \sin(\sin(x^3))\cos(x^3) \, dx$

$-\dfrac{1}{3}\cos\left(\sin(x^3)\right) + C$

14. $\displaystyle\int \left(\tan^2(x) + \sin^2(x) + \sec^2(x) + \csc^2(x) + \cot^2(x) + \cos^2(x)\right) \, dx$

$-x + 2\tan x - 2\cot x + C$

15. $\displaystyle\int \frac{e^x - 1}{e^x + 1}\, dx$

$2\ln(e^x + 1) - x + C$

16. $\displaystyle\int \frac{dx}{x^8(x^2+1)}$

$$\frac{1}{x^8(x^2+1)} = \frac{1}{x^8} - \frac{1}{x^6} + \frac{1}{x^4} - \frac{1}{x^2} + \frac{1}{x^2+1}$$

$$I = -\frac{1}{7x^7} + \frac{1}{5x^5} - \frac{1}{3x^3} + \frac{1}{x} + \arctan x + C$$

17. $\displaystyle\int \exp(5\ln x)\, dx$

$x^6/6 + C$

18. $\displaystyle\int \frac{\ln x}{x^a}\, dx, \qquad a \neq 1$

$$-\frac{1}{a-1}\frac{\ln x}{x^{a-1}} - \frac{1}{(a-1)^2}\frac{1}{x^{a-1}}$$

19. $\displaystyle\int e^{-e^x} e^x\, dx$

$-e^{-e^x} + C$

20. $\displaystyle\int_0^u \frac{x}{1 + \frac{x}{1+\frac{x}{1+\frac{x}{1+\cdots}}}}\, dx, \qquad$ Note: $0 < u < 1/4$.

solve $f(x) = x/(1+f(x)) \Rightarrow f(x) = (1+4x)^{1/2}/2 - 1/2$

indefinite integral: $I = \dfrac{1}{12}(1+4x)^{3/2} - x/2 + C$

$$I = \frac{1}{12}(1+4u)^{3/2} - u/2 - \frac{1}{12}$$