SwInBee 2019

Name:

Instructions

1. Duration: 1 hour.
2. 1 point for each correct answer, -1 point for incorrect guesses. (Guesses are identified at the marker's discretion; basically we want to discourage writing "π" for each integral that you can't figure out.)
3. Materials allowed are pens, pencils, and the TI-30XB "green" calculator (preferred) or an equivalent non-programmable calculator. We'll supply paper for rough working.
4. All integrals will be supplied as definite integrals, and on your solution sheet you must write your answers as numerical values which are accurate to at least 6 decimal places.
5. In the event of papers achieving the same score, the tie-breaker will be the order of submission, with earlier papers ranked higher.

Integrals

1. $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2 x \, dx = \left[\frac{x}{2} - \frac{1}{4}\sin(2x)\right]_0^{\frac{\pi}{4}} = -\frac{1}{4} + \frac{\pi}{8} = 0.1426990817\cdots$

2. $\displaystyle\int_0^2 \sqrt{4-x^2}\, dx = \left[\frac{1}{2}x\sqrt{4-x^2} + 2\arcsin\left(\frac{x}{2}\right)\right]_0^2 = \pi = 3.141592654\cdots$

3. $\displaystyle\int_0^5 \frac{x}{x^2+5}\, dx = \left[\frac{1}{2}\log\left(x^2+5\right)\right]_0^5 = \frac{1}{2}\ln 6 = 0.8958797346\cdots$

4. $\displaystyle\int_0^9 \frac{1}{\sqrt{1+\sqrt{x}}}\, dx = \frac{16}{3} = 5.333333333\cdots$
   Let $t^2 = 1+\sqrt{x}$.

5. $\displaystyle\int_0^1 \frac{x\arcsin(x^2)}{\sqrt{1-x^4}}\, dx = \left[\frac{1}{4}\arcsin^2\left(x^2\right)\right]_0^1 = \frac{\pi^2}{16} = 0.6168502751\cdots$

6. $\displaystyle\int_0^\infty \frac{1}{1+x^2}\, dx = [\arctan x]_0^\infty = \frac{\pi}{2} = 1.570796327\cdots$

7. $\displaystyle\int_0^1 x^5 e^{x^3}\, dx = \left[\frac{1}{3}e^{x^3}(x^3-1)\right]_0^1 = \frac{1}{3} = 0.33333333333\cdots$

8. $\displaystyle\int_2^3 \frac{1}{x^2-1}\,dx = \left[\frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)\right]_2^3 = 0.2027325541\cdots$

9. $\displaystyle\int_{-\pi}^{\pi} x^4 \sin x\,dx = 0$ (n.b.: integrand is odd)

10. $\displaystyle\int_0^{\pi} \sin 2x\,\cos 3x\,dx = \left[\frac{1}{2}\cos x - \frac{1}{10}\cos 5x\right]_0^{\pi} = -0.8$

11. $\displaystyle\int_0^1 \frac{1-x}{(1+x)^2}\,dx = \left[-\frac{2}{1+x} - \ln|1+x|\right]_0^1 = 1 - \ln 2 = 0.3068528194\cdots$

12. $\displaystyle\int_0^{\infty} \frac{1-x}{(1+x)^3}\,dx = \left[\frac{x}{(1+x)^2}\right]_0^{\infty} = -\frac{1}{4} = -0.25$

13. $\displaystyle\int_1^3 \ln^4 x\,dx = \left[24x - 24x\ln x + 12x\ln^2 x - 4x\ln^3 x + x\ln^4 x\right]_1^3 = 0.8086276654\cdots$

14. $\displaystyle\int_0^{\frac{\pi}{2}} \cos x\,\sin(\sin x)\cos(\cos(\sin x))\,dx = \left[-\sin(\cos(\sin x))\right]_0^{\frac{\pi}{2}} = \sin(1) - \sin(\cos(1)) = 0.3270757263\cdots$

15. $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\,\cos x\,\cosh(3x)\,dx = \left[\frac{3}{26}\sin(2x)\sinh(3x) - \frac{1}{13}\cos(2x)\cosh(3x)\right]_0^{\frac{\pi}{2}} = 4.358721607\cdots$

16. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^2 x\; e^{\sin x}}{\tan x}\,dx = \left[e^{\sin x}(-1+\sin x)\right]_0^{\frac{\pi}{2}} = 1$

17. $\displaystyle\int_0^{\frac{1}{6}} \left(1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + \cdots\right)\,dx = \left[x + x^2 + x^3 + x^4 + x^5 + x^6 + \cdots\right]_0^{\frac{1}{6}} =$
$\left[1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + \cdots\right]_0^{\frac{1}{6}}$
$\displaystyle= \left[\frac{1}{1-x}\right]_0^{\frac{1}{6}} = \frac{1}{5} = 0.2$
Geometric series! $\displaystyle\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

18. $\displaystyle\int_0^{\frac{1}{6}} \left(1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + \cdots\right)\,dx = \left[x - x^2 + x^3 - x^4 + x^5 - x^6 + \cdots\right]_0^{\frac{1}{6}} =$
$\left[-1 + x - x^2 + x^3 - x^4 + x^5 - x^6 + \cdots\right]_0^{\frac{1}{6}}$

$$= \left[-\frac{1}{1+x}\right]_0^{\frac{1}{6}} = \frac{1}{7} = 0.142857142857142857\cdots$$

Geometric series! $\quad \dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$

19. $\displaystyle\int_0^1 \sqrt{x+\sqrt{x+\sqrt{x+\sqrt{\cdots}}}}\,dx = \int_0^1 \frac{1+\sqrt{1+4x}}{2}\,dx = \left[\frac{1}{2}x + \frac{1}{12}(1+4x)^{3/2}\right]_0^1$

$= \dfrac{5}{12}\left(1+\sqrt{5}\right) = 1.348361657\cdots$

Let $y =$integrand, then we have $\quad y = \sqrt{x+y} \Rightarrow y = (1+\sqrt{1+4x})/2$

20. $\displaystyle\lim_{n\to\infty} \frac{\int_1^n x^n \ln x\,dx}{n^n \ln n} = 1$