SwInBee 2018

Name:

Instructions

1. Duration: 1 hour.
2. No materials allowed besides pens and pencils. Paper will be supplied for rough working.
3. No partial marks awarded. This includes the "+ C" for indefinite integrals: if an appropriate constant is not included then you will get zero.
4. A prize of $25 will be given for solution of the prize question, with the tie-breaker for multiple correct answers being the total number of points.

Integrals

1. $\displaystyle\int \sqrt{x} e^{x\sqrt{x}}\,dx = \frac{2}{3} e^{x^{3/2}} + C$

2. $\displaystyle\int \frac{1}{\sqrt{x}} \left(\ln \sqrt{x} + \frac{1}{x} \right) dx = \frac{-2(1+x) + x \ln x}{\sqrt{x}} + C$

3. $\displaystyle\int \sinh x \arctan(\sinh x)\,dx = \arctan(\sinh x) \cosh x - x + C$

4. $\displaystyle\int \frac{dx}{x^3 + x} = \ln x - \frac{1}{2} \ln(1 + x^2) + C$

5. $\displaystyle\int \frac{1 + e^x}{e^x - 1}\,dx = 2 \ln(1 - e^x) - x + C$

6. $\displaystyle\int e^x \cos(\cos(e^x)) \sin(e^x)\,dx = -\sin(\cos e^x) + C$

7. $\displaystyle\int \frac{x^3}{1 + x^2}\,dx = \frac{1}{2} x^2 - \frac{1}{2} \ln(1 + x^2) + C$

8. $\displaystyle\int \ln\left(\frac{1+x}{1-x} \right) dx = \ln(1 - x) + \ln(1 + x) + x \ln\left(\frac{1+x}{1-x} \right) + C$

9. $\displaystyle\int \frac{1-\cos x}{\sin\frac{x}{2}}\,dx = -4\cos\left(\frac{x}{2}\right)+C$

10. $\displaystyle\int \frac{x^2}{2}\ln\left(\frac{2}{x^2}\right)\,dx = \frac{1}{9}x^3+\frac{1}{6}x^3\ln\left(\frac{2}{x^2}\right)+C$

11. $\displaystyle\int \frac{dx}{x^2-1} = \frac{1}{2}\ln(1-x)-\frac{1}{2}\ln(1+x)+C$

12. $\displaystyle\int \frac{\sinh x\cos(\cosh x)}{\sin^2(\cosh x)}\,dx = -\frac{1}{\sin(\cosh x)}+C$

13. $\displaystyle\int \frac{dx}{\sqrt{7-x^2}} = \arcsin\left(\frac{x}{\sqrt{7}}\right)+C$

14. $\displaystyle\int_{-2\pi}^{2\pi}(x^3\cos 4x-(x^4+x^2+1)\sin 3x)\,dx = 0$ n.b.: the integrand is odd.

15. $\displaystyle\int x^8e^{x^3}\,dx = \frac{1}{3}e^{x^3}(2-2x^3+x^6)+C$

16. $\displaystyle\int_0^{2018}|\sin(2018\pi x)|\,dx = \frac{4036}{\pi}$

17. $\displaystyle\int \frac{x^2-3x+2}{(x+1)^3}\,dx = \frac{2+5x}{(1+x)^2}+\ln(1+x)+C$

18. $\displaystyle\int (x(x(x(x(\cdots)^{1/2})^{1/2})^{1/2})^{1/2})^{1/2}\,dx = \frac{1}{2}x^2+C$
n.b.: the geometric series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=1$

19. $\displaystyle\int \cos^8 x\,dx = \frac{35}{128}x+\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)+\frac{1}{96}\sin(6x)+\frac{1}{1024}\sin(8x)+C$

20. **Prize question!** $\displaystyle\int_{-\infty}^{\infty}e^{-3x^2}\,dx = \sqrt{\frac{\pi}{3}}$