SwInBee

Practice and revision

Preparing for the Bee

The qualifying round is **20 integrals in 50 minutes**, answers written on the paper. There are **no partial marks**, and that includes the "$+C$": an indefinite integral without a constant of integration scores zero. Finalists then work integrals on a whiteboard.

This handout is meant as a refresher, not as a comprehensive resource. It includes examples from past SwInBee competitions.

Before the day

- Memorise the standard integrals and various identities.
- Attempt old test papers.
- Study this document and attempt the exercises.

On the day

- **Scan the whole paper** and do the easy ones first.
- **Write the $+C$.** Every time, on every indefinite integral.
- **Check your answers by differentiating.** (If you have time.)
- If you're stuck, throw ideas at the integral – maybe there's a simple insight or trick that can make things simpler.

Solutions to exercises are provided separately.

1 Useful definitions and identities

Trigonometric definitions

$$\tan x = \frac{\sin x}{\cos x}, \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}, \qquad \sec x = \frac{1}{\cos x}, \qquad \csc x = \frac{1}{\sin x}.$$

Pythagorean identities

$$\sin^2 x + \cos^2 x = 1, \qquad 1 + \tan^2 x = \sec^2 x, \qquad 1 + \cot^2 x = \csc^2 x.$$

Addition formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B, \qquad \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B,$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}.$$

Multiple angles

$$\sin 2x = 2\sin x\cos x, \qquad \tan 2x = \frac{2\tan x}{1-\tan^2 x},$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x,$$
$$\sin 3x = 3\sin x - 4\sin^3 x, \qquad \cos 3x = 4\cos^3 x - 3\cos x.$$

Also:

$$\sin^2 x = \tfrac{1}{2}\left(1-\cos 2x\right), \qquad \cos^2 x = \tfrac{1}{2}\left(1+\cos 2x\right).$$

Products to sums

$$\sin A\cos B = \tfrac{1}{2}\left[\sin(A+B)+\sin(A-B)\right], \qquad \sin A\sin B = \tfrac{1}{2}\left[\cos(A-B)-\cos(A+B)\right],$$
$$\cos A\cos B = \tfrac{1}{2}\left[\cos(A-B)+\cos(A+B)\right].$$

Sums to products

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),$$
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right).$$

The half-angle (Weierstrass) substitution

With $t = \tan\frac{x}{2}$, every trigonometric function becomes rational:

$$\sin x = \frac{2t}{1+t^2}, \qquad \cos x = \frac{1-t^2}{1+t^2}, \qquad \tan x = \frac{2t}{1-t^2}, \qquad dx = \frac{2\,dt}{1+t^2}.$$

Hyperbolic functions

$$\cosh x = \frac{e^x+e^{-x}}{2}, \qquad \sinh x = \frac{e^x-e^{-x}}{2}, \qquad \tanh x = \frac{\sinh x}{\cosh x},$$

$$\cosh^2 x - \sinh^2 x = 1, \qquad 1-\tanh^2 x = \operatorname{sech}^2 x, \qquad \sinh 2x = 2\sinh x\cosh x,$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x,$$
$$\frac{d}{dx}\sinh x = \cosh x, \qquad \frac{d}{dx}\cosh x = \sinh x.$$

The inverses are logarithms:

$$\sinh^{-1} x = \ln\left(x+\sqrt{x^2+1}\right), \qquad \cosh^{-1} x = \ln\left(x+\sqrt{x^2-1}\right), \qquad \tanh^{-1} x = \tfrac{1}{2}\ln\frac{1+x}{1-x}.$$

Euler’s formula

$$e^{ix} = \cos x + i\sin x, \qquad \cos x = \frac{e^{ix}+e^{-ix}}{2}, \qquad \sin x = \frac{e^{ix}-e^{-ix}}{2i},$$

and de Moivre's theorem, $(\cos x + i \sin x)^n = \cos nx + i \sin nx$.

Exponentials and logarithms

$$a^x = e^{x \ln a}, \qquad \log_a x = \frac{\ln x}{\ln a}, \qquad \ln(xy) = \ln x + \ln y, \qquad \ln\left(x^p\right) = p \ln x, \qquad e^{\ln x} = x.$$

Algebra

$$a^2 - b^2 = (a-b)(a+b), \qquad a^3 \pm b^3 = (a \pm b)\left(a^2 \mp ab + b^2\right),$$
$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + \left(c - \frac{b^2}{4}\right), \qquad 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} \quad (|x| < 1).$$

2 Standard integrals

In the table, $a > 0$, and the $+C$ is omitted **but must be included in your answer**.

$\int x^n \, dx$	$\dfrac{x^{n+1}}{n+1}$	$(n \neq -1)$	$\int \sinh x \, dx$	$\cosh x$	
$\int \dfrac{dx}{x}$	$\ln \lvert x \rvert$		$\int \cosh x \, dx$	$\sinh x$	
$\int e^x \, dx$	e^x		$\int \dfrac{dx}{a^2 + x^2}$	$\dfrac{1}{a} \arctan \dfrac{x}{a}$	
$\int \sin x \, dx$	$-\cos x$		$\int \dfrac{dx}{\sqrt{a^2 - x^2}}$	$\arcsin \dfrac{x}{a}$	$(\lvert x \rvert < a)$
$\int \cos x \, dx$	$\sin x$		$\int \dfrac{dx}{\sqrt{x^2 + a^2}}$	$\sinh^{-1} \dfrac{x}{a}$	
$\int \sec^2 x \, dx$	$\tan x$		$\int \dfrac{dx}{\sqrt{x^2 - a^2}}$	$\cosh^{-1} \dfrac{x}{a}$	$(x > a)$
$\int \csc^2 x \, dx$	$-\cot x$		$\int \dfrac{dx}{a^2 - x^2}$	$\dfrac{1}{a} \tanh^{-1} \dfrac{x}{a}$	$(\lvert x \rvert < a)$
$\int \sec x \tan x \, dx$	$\sec x$				

The last three written as logarithms:

$$\sinh^{-1} \frac{x}{a} = \ln\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right), \qquad \cosh^{-1} \frac{x}{a} = \ln\left(\frac{x + \sqrt{x^2 - a^2}}{a}\right),$$
$$\frac{1}{a} \tanh^{-1} \frac{x}{a} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|.$$

Additional notes:

- **Linear arguments:** If $\int f(x)\, dx = F(x) + C$ then $\displaystyle\int f(ax+b)\, dx = \frac{1}{a} F(ax+b) + C$.
- **Complete the square:** $x^2 + 4x + 13 = (x+2)^2 + 9$, so $\int \frac{dx}{x^2+4x+13} = \frac{1}{3} \arctan \frac{x+2}{3} + C$.

3 The substitution method

The method. From the chain rule we get

$$\int f\left(g(x)\right) g'(x)\, dx = \int f(u)\, du, \quad \text{where } u = g(x).$$

For a definite integral, change the limits as well: if x runs from a to b, then u runs from $g(a)$ to $g(b)$, and there is no need to substitute back at the end.

When to try it. There are three common situations.

1. **The derivative of an inner function is present.** If one function is composed inside another, and the derivative of the inner function appears as a factor (up to a constant), take u to be the inner function.
2. **A root.** Set t equal to the root, or set $x = t^n$ where n removes all of the fractional powers. The integrand usually becomes a rational function.
3. **A quadratic under a square root.** Use a trigonometric or hyperbolic substitution:

   $$\sqrt{a^2 - x^2}: \; x = a\sin\theta, \qquad \sqrt{a^2 + x^2}: \; x = a\sinh t, \qquad \sqrt{x^2 - a^2}: \; x = a\cosh t.$$

   Each substitution turns the expression under the root into a perfect square. If the quadratic is not in one of these forms, complete the square first.

Worked example 3.1 SwInBee 2022

$$\int \frac{x}{1+x^4}\, dx$$

Spotting it: Writing $x^4 = \left(x^2\right)^2$, the integrand is $\frac{x}{1+\left(x^2\right)^2}$. The inner function is x^2, and its derivative $2x$ appears in the numerator up to a constant factor. A factor of x multiplying a function of x^2 often suggests the substitution $u = x^2$.
With $u = x^2$ we have $du = 2x\, dx$, so $x\, dx = \frac{1}{2} du$:

$$\int \frac{x}{1+x^4}\, dx = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\arctan u + C = \frac{1}{2}\arctan\left(x^2\right) + C.$$

Answer: $\frac{1}{2}\arctan\left(x^2\right) + C$

Worked example 3.2 SwInBee 2024

$$\int \frac{dx}{1+\sqrt{x}}$$

Spotting it: There is no inner derivative to use here. However, the square root is the only thing making the integral difficult, so we remove it with the substitution $x = t^2$.

With $x = t^2$ (so $t = \sqrt{x}$) and $dx = 2t\,dt$:

$$\int \frac{dx}{1+\sqrt{x}} = \int \frac{2t}{1+t}\,dt = 2\int \left(1 - \frac{1}{1+t}\right) dt = 2t - 2\ln(1+t) + C,$$

where we used long division in the middle step. Substituting back,

$$\int \frac{dx}{1+\sqrt{x}} = 2\sqrt{x} - 2\ln\left(1+\sqrt{x}\right) + C.$$

Answer: $2\sqrt{x} - 2\ln\left(1+\sqrt{x}\right) + C$

If several different roots appear, take $x = t^n$ where n is the lowest common multiple of their orders.

Worked example 3.3 SwInBee 2024

$$\int \frac{\sqrt{x^2-1}}{x}\,dx$$

Spotting it: The integrand contains $\sqrt{x^2-a^2}$ with $a = 1$. We want a substitution that makes $x^2 - 1$ a perfect square, and since $\sec^2\theta - 1 = \tan^2\theta$, we try $x = \sec\theta$. (The substitution $x = \cosh t$ also works.)

With $x = \sec\theta$ we have $dx = \sec\theta\tan\theta\,d\theta$ and $\sqrt{x^2-1} = \tan\theta$:

$$\int \frac{\sqrt{x^2-1}}{x}\,dx = \int \frac{\tan\theta}{\sec\theta}\,\sec\theta\tan\theta\,d\theta = \int \tan^2\theta\,d\theta = \int \left(\sec^2\theta - 1\right) d\theta = \tan\theta - \theta + C.$$

Since $\tan\theta = \sqrt{x^2-1}$ and $\theta = \arctan\sqrt{x^2-1}$,

$$\int \frac{\sqrt{x^2-1}}{x}\,dx = \sqrt{x^2-1} - \arctan\sqrt{x^2-1} + C \qquad (x > 1).$$

Answer: $\sqrt{x^2-1} - \arctan\sqrt{x^2-1} + C$

or $\sqrt{x^2-1} - \arccos\left(\dfrac{1}{x}\right) + C$

The two forms are equal, since $\arctan\sqrt{x^2-1} = \arccos(1/x)$ for $x > 1$. Answers from trigonometric substitutions can look quite different and still be correct, so differentiate to check if you are unsure.

Exercises

3.1 $\displaystyle\int \sin x\,\cos(\cos x)\sin(\sin(\cos x))\,dx$ SwInBee 2022

3.2 $\displaystyle\int_0^9 \frac{dx}{\sqrt{1+\sqrt{x}}}$ SwInBee 2019

3.3 $\displaystyle\int \frac{dx}{\sqrt{x^2-2024}}$ SwInBee 2024

3.4 $\displaystyle\int_0^2 \sqrt{4-x^2}\,dx$ SwInBee 2019

4 Partial fraction decomposition

The method. Any rational function, $P(x)/Q(x)$ with P and Q polynomials, can be integrated using partial fractions. The steps are:

1. **Divide first if necessary.** If $\deg P \geq \deg Q$, use long division to write P/Q as a polynomial plus a proper fraction. For example, $\frac{x+2025}{x-2025} = 1 + \frac{4050}{x-2025}$.
2. **Factor the denominator** into linear factors and irreducible quadratics.
3. **Write down the partial fractions** using the templates below, and find the constants.

Templates

factor of the denominator	partial fractions
$x-a$	$\dfrac{A}{x-a}$
$(x-a)^k$	$\dfrac{A_1}{x-a} + \dfrac{A_2}{(x-a)^2} + \cdots + \dfrac{A_k}{(x-a)^k}$
x^2+bx+c (irreducible)	$\dfrac{Ax+B}{x^2+bx+c}$
$\left(x^2+bx+c\right)^k$	$\dfrac{A_1x+B_1}{x^2+bx+c} + \cdots + \dfrac{A_kx+B_k}{(x^2+bx+c)^k}$

A quadratic x^2+bx+c is irreducible when $b^2-4c<0$. If it factors, use the linear templates instead.

Finding the constants. Multiply through by the denominator, then either compare coefficients or substitute convenient values of x (the roots of the factors are usually the best choice).

Integrating the pieces.

$$\int \frac{A\,dx}{x-a} = A\ln|x-a| + C, \qquad \int \frac{A\,dx}{(x-a)^k} = \frac{-A}{(k-1)\,(x-a)^{k-1}} + C \quad (k>1).$$

For a quadratic denominator, split the numerator into a multiple of the derivative of the denominator, which integrates to a logarithm, plus a constant, which gives an $\arctan$ after completing the square.

Worked example 4.1 SwInBee 2024

$$\int \frac{3x+2}{x^2+3x+2}\,dx$$

The denominator factors as $(x+1)(x+2)$, so we write $\frac{3x+2}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$. Multiplying through by $(x+1)(x+2)$ gives

$$3x+2 = A(x+2) + B(x+1).$$

Setting $x=-1$ gives $-1=A$, and setting $x=-2$ gives $-4=-B$, so $A=-1$ and $B=4$. Then

$$\int \frac{3x+2}{x^2+3x+2}\,dx = \int \left(\frac{4}{x+2} - \frac{1}{x+1}\right) dx = 4\ln|x+2| - \ln|x+1| + C.$$

Answer: $4\ln|x+2| - \ln|x+1| + C$

Worked example 4.2 SwInBee 2025

$$\int \frac{x+4}{(x-1)^2}\,dx$$

There is a repeated linear factor, so the template is $\frac{A}{x-1} + \frac{B}{(x-1)^2}$. Here it is quicker to write the numerator as $x+4 = (x-1)+5$, which gives the decomposition directly:

$$\frac{x+4}{(x-1)^2} = \frac{1}{x-1} + \frac{5}{(x-1)^2}, \qquad \int \frac{x+4}{(x-1)^2}\,dx = \ln|x-1| - \frac{5}{x-1} + C.$$

Answer: $\ln|x-1| - \dfrac{5}{x-1} + C$

Only the $\frac{1}{x-1}$ term gives a logarithm. The $\frac{1}{(x-1)^2}$ term does not.

Worked example 4.3 SwInBee 2018

$$\int \frac{dx}{x^3+x}$$

The denominator factors as $x\left(x^2+1\right)$, where x^2+1 is irreducible, so we write $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$. Multiplying through gives $1 = A\left(x^2+1\right) + (Bx+C)\,x$. Setting $x=0$ gives $A=1$, and comparing the coefficients of x^2 and x gives $A+B=0$ and $C=0$, so $B=-1$:

$$\int \frac{dx}{x\left(x^2+1\right)} = \int \left(\frac{1}{x} - \frac{x}{x^2+1}\right) dx = \ln|x| - \frac{1}{2}\ln\left(1+x^2\right) + C.$$

Answer: $\ln|x| - \dfrac{1}{2}\ln\left(1+x^2\right) + C$

The second term gives a logarithm because its numerator is a multiple of the derivative of its denominator. A constant numerator would have given an $\arctan$ instead.

Exercises

4.1 $\displaystyle\int \frac{dx}{x^2+5x+6}$ SwInBee 2022

4.2 $\displaystyle\int \frac{2x^3}{5+3x^2}\,dx$ SwInBee 2024

4.3 $\displaystyle\int \frac{4x+1}{1+x^2}\,dx$ SwInBee 2024

4.4 $\displaystyle\int \frac{dx}{x^3\left(x^2+1\right)}$ SwInBee 2025

5 Integration by parts

The method. From the product rule we get

$$\int u\,dv = uv - \int v\,du \quad \text{or} \quad \int f'g = fg - \int fg'$$

The key idea is to choose u (or g) so that the new integral is easier than the old one. We want this function to become simpler when differentiated, while v (or f) should not become more complicated when integrated.

Functions that become simpler when differentiated include logarithms and inverse trigonometric functions.

When to try it. A product of two unrelated kinds of function, e.g. a polynomial multiplied by an exponential, a polynomial multiplied by a trig function, anything multiplied by logarithm, where there are no obvious substitutions. Sometimes an awkward function like $\ln x$, $\arctan x$, $\arcsin x$ can be treated as u, with $dv = dx$ (alternatively, set the awkward function to g, and set $f(x) = x$).

Worked example 5.1 SwInBee 2025 (finals)

$$\int x \sin 2x \, dx$$

Spotting it: A polynomial times a trig function, and nothing inside $\sin 2x$ whose derivative appears elsewhere. Take $u = x$: differentiating it gives 1 and the x disappears, whereas $\sin 2x$ only ever cycles round to $\pm\cos 2x$ and $\pm\sin 2x$ no matter what you do to it.

With $u = x$ and $dv = \sin 2x \, dx$, so $du = dx$ and $v = -\frac{1}{2}\cos 2x$:

$$\int x \sin 2x \, dx = -\frac{x\cos 2x}{2} + \frac{1}{2}\int \cos 2x \, dx = -\frac{x\cos 2x}{2} + \frac{\sin 2x}{4} + C.$$

Answer: $\dfrac{\sin 2x}{4} - \dfrac{x\cos 2x}{2} + C$

Check it: differentiating gives $\frac{\cos 2x}{2} - \frac{\cos 2x}{2} + x\sin 2x = x \sin 2x$.

Worked example 5.2 SwInBee 2022

$$\int \ln\left(1 + x^2\right) dx$$

Spotting it: There is only one function here, so there is apparently nothing to split, so we try $dv = dx$. Parts then replaces the logarithm by its derivative, and the derivative of a logarithm is rational.

With $u = \ln(1 + x^2)$ and $dv = dx$, so $du = \frac{2x}{1+x^2}dx$ and $v = x$:

$$\int \ln\left(1 + x^2\right) dx = x\ln\left(1 + x^2\right) - \int \frac{2x^2}{1 + x^2}\, dx.$$

We need to reduce the order of the numerator, so divide: $\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$, and finally we get:

$$\begin{aligned}\int \ln\left(1 + x^2\right) dx &= x\ln\left(1 + x^2\right) - 2\int\left(1 - \frac{1}{1 + x^2}\right) dx \\ &= x\ln\left(1 + x^2\right) - 2x + 2\arctan x + C.\end{aligned}$$

Answer: $x\ln\left(1 + x^2\right) - 2x + 2\arctan x + C$

The same trick handles $\int \arctan x \, dx$, $\int \arcsin x \, dx$ and $\int \ln x \, dx$.

Worked example 5.3 SwInBee 2023 and 2025

$\int e^x \sin x \, dx$

Spotting it: Neither factor becomes simpler when differentiated, so one application of parts will not finish the job. However, after two applications the original integral reappears, and we can then solve for it.

Call the integral I. With $u = e^x$ and $dv = \sin x \, dx$:

$$I = \int e^x \sin x \, dx = -e^x \cos x + \int e^x \cos x \, dx.$$

Now apply parts to the new integral, with the same choice of u (swapping the roles of u and dv would just undo the first step):

$$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx = e^x \sin x - I.$$

So $I = -e^x \cos x + e^x \sin x - I$, and therefore

$$2I = e^x (\sin x - \cos x), \qquad I = \frac{e^x}{2} (\sin x - \cos x) + C.$$

Answer: $\frac{1}{2} e^x (\sin x - \cos x) + C$

The same approach works for any integral of the form $e^{ax} \sin bx$ or $e^{ax} \cos bx$.

Exercises

5.1 $\int x^2 \ln x \, dx$ SwInBee 2022

5.2 $\int \frac{4x}{e^{2x+3}} \, dx$ SwInBee 2024

5.3 $\int x^2 \arcsin x \, dx$ SwInBee 2023

5.4 $\int (\ln x)^3 \, dx$ SwInBee 2022

6 Tricky integrals

The method. Some integrals become much easier with the right insight, and some cannot be done without one. If an integral looks hard, it is worth checking the following:

- **Does the integrand simplify?** Try expanding, factorising, cancelling, or applying an identity.
- **Is the integrand odd, on an interval symmetric about 0?** If so, the integral is 0.
- **Is the integrand the result of the product or quotient rule?** This is worth considering when the integrand is a sum of terms that cannot be integrated separately.
- **Does the substitution $x \mapsto a + b - x$ help?** Since $\int_a^b f(x) \, dx = \int_a^b f(a + b - x) \, dx$, adding the two versions sometimes gives an integrand that simplifies. For example, since

$8x^3 - 12x^2 + 6x - 1 = (2x-1)^3$, the substitution $x \mapsto 1 - x$ changes the sign of the integrand in $\int_0^1 \frac{\arcsin\left(8x^3-12x^2+6x-1\right)}{x^2-x+1}\,dx$, so this integral is 0.

- **Is it a series?** For example, a geometric or telescoping series, or a Taylor series. Sum the series first, then integrate.
- **Is it self-similar?** An expression like $\sqrt{x\sqrt{x\sqrt{x\cdots}}}$ can be simplified by calling the whole expression y and finding an equation that y satisfies.

If none of these help, move on, and come back later if there is time.

Worked example 6.1 SwInBee 2025

$$\int \cos^4 x - \sin^4 x\,dx$$

Spotting it: Integrating fourth powers of sin and cos directly would take a lot of work. However, the integrand is a difference of two squares, $A^2 - B^2 = (A-B)(A+B)$, with $A = \cos^2 x$ and $B = \sin^2 x$.

$$\cos^4 x - \sin^4 x = \left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right) = \cos 2x,$$

using $\cos^2 x - \sin^2 x = \cos 2x$ and $\cos^2 x + \sin^2 x = 1$. So

$$\int \cos 2x\,dx = \frac{\sin 2x}{2} + C.$$

Answer: $\frac{1}{2}\sin 2x + C$ or $\sin x \cos x + C$

The two forms are equal by the double-angle formula.

Worked example 6.2 SwInBee 2022

$$\int_{-\pi}^{\pi} x^3 \cos x\, \sin^4 x\,dx$$

Spotting it: Finding an antiderivative would take a lot of work. However, the limits are symmetric about 0, so we check whether the integrand is odd.

The factor x^3 is odd, while $\cos x$ and $\sin^4 x$ are both even, so the integrand is odd. The contributions from $[-\pi, 0]$ and $[0, \pi]$ cancel:

$$\int_{-\pi}^{\pi} x^3 \cos x \sin^4 x\,dx = 0.$$

Answer: 0

Worked example 6.3 SwInBee 2022

$$\int \frac{1}{\ln x} - \frac{1}{(\ln x)^2}\,dx$$

Spotting it: Neither term can be integrated on its own in terms of elementary functions. When this happens, the integrand may be the derivative of a product or a quotient. The powers of $\ln x$ in the denominators suggest trying $x/\ln x$.

Differentiating $x/\ln x$ using the quotient rule,

$$\frac{d}{dx}\left[\frac{x}{\ln x}\right] = \frac{\ln x \cdot 1 - x \cdot \frac{1}{x}}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2} = \frac{1}{\ln x} - \frac{1}{(\ln x)^2},$$

which is the integrand. Hence

$$\int \left(\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right) dx = \frac{x}{\ln x} + C.$$

Answer: $\dfrac{x}{\ln x} + C$

Guessing an answer and checking it by differentiation is a perfectly valid method.

Exercises

6.1 $\displaystyle\int \exp\left(5\ln x\right) dx$ SwInBee 2023

6.2 $\displaystyle\int_{-\pi}^{\pi} x^{11} \sin^3\left(x^2\right) dx$ SwInBee 2024

6.3 $\displaystyle\int_0^{1/6} \left(1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + \cdots\right) dx$ SwInBee 2019

6.4 $\displaystyle\int_0^1 \frac{x^4\left(1-x\right)^4}{1+x^2}\, dx$ SwInBee 2024 and 2025